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A CHAOTIC AGRICULTURE/AGRI-INDUSTRY RATIO GROWTH MODEL

Vesna D. Jablanovic[1]

Abstract

Chaos theory , as a set of ideas , attempts to reveal structure in aperiodic, unpredictable dynamic systems. Chaos embodies three important principles: (i) extreme sensitivity to initial conditions; (ii) cause and effect are not proportional; and (iii) nonlinearity.

The basic aim of this paper is to provide a relatively simple the agriculture/agri-industry ratio growth model that is capable of generating stable equilibria, cycles, or chaos depending on parmeter values.

A key hypothesis of this work is based on the idea that the coefficient $\pi = \gamma + 1$ *plays a crucial role in explaining local stability of the agriculture/agri-industry ratio,where* γ *is a suitable parameter.*

Key words: *chaos, agriculture, agri-industry, growth, model.*

Introduction

Input-output analysis, developed by W.W. Leontief, is used to study the relations between economic sectors. Leontief 's concern focused on how economic systems were structured, the way an economic sectors interrelate and mutually influence one another.

Input-output analysis as a basic method of quantitative economics observes various economic sectors as a series of inputs of source materials (or services) and outputs of finished or semi-finished goods (or services).

Commodities (or services) are produced by economic sectors (e.g. the agricultural sector) and they serve as inputs in other sectors in order to produce their final products (or services) also called outputs (e.g. manufacturing industry, agri-industry, turism , trade,…). Intermediate demand refers to inter-industry transactions, i.e. goods and services bought by firms from other firms and used up in current production.

The outputs are delivered to the final demand sector that comprises purchases

1 Vesna D. Jablanovic, Ph.D., Associate Professor, Faculty of Agriculture, University of Belgrade, Nemanjina 6, Belgrade, Serbia, vesnajab@ptt.rs

by individuals for consumption, by firms for investment, by government for government purchases , and by foreigners for exports.

Also, firms use primary inputs (factors of production) which are bought from individuals (wages and salaries as payments for labour services, interests paid on borrowing, rent paid for the use of equipment, building and land, profits paid for the entrepreneurship).

The output of any industry (say, the agricultural sector) is needed as an input in many other industries, or even for that industry itself; therefore the equilibrium level of the observed sector (say, the agricultural sector) output will depend on the input requirements of all the n industries. In turn, the output of many other industries will enter into the agriculture as inputs. The equilibrium outputs of the other sectors will depend partly upon the input requirements of the observed sector (say, agriculture).

If the agricultural sector , for example, increased its production by one unit, then exists the following:

i) first round, direct effects on the industries that supply the agricultural sector with inputs; and

ii) secondary round, indirect effects, since these supplier industries themselves require additional inputs for their production, in order to meet the additional demand coming from the agricultural sector production system.

Technical coefficient plays important role if we take into account these effects. Technical (input) coefficient represents the "recipe" for production of the economic sector. The coefficient indicates the linkages between inputs and outputs and identifies the percentage or portion of the total inputs of a sector required to be purchased from another sector. These ratios of inputs to outputs reflects production technologies at a given point in time.

The assumption that production coefficients remained constant for extended periods is not in accordance with the possibility that factors of production, were substituted for one another as their relative prices changed .

The existence of stable technical coefficients within a longer term forecast is tenuous. The dynamics of the technical coefficients (relative input price changes, the appearance of new industries during the projection period, and the effects of technological change,…) require dynamic approach.

This paper's concern focused on the dynamics of the particular technical coefficient, the agriculture/agri-industry ratio. Namely, the output of agriculture is input of agri-industry. It is assumed that the agriculture/agri-industry ratio is separated from an input-output model.

Chaos theory is used to prove that chaotic fluctuations can indeed arise in completely deterministic models. Chaos theory reveals structure in aperiodic, dynamic systems. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983,1997,), Grandmont (1985), Goodwin (1990), Medio (1993,1996), Medio, A. and Lines, M (2004), Lorenz (1993), Shone, R.(1999), Jablanovic (2010), among many others.

The basic aim of this paper is to provide a relatively simple the agriculture/ agri-industry ratio growth model that is capable of generating stable equilibria, cycles, or chaos depending on parmeter values.

The model

The interdependence between agriculture (as an input sector) and agri-industry (as an output sector) can be analyzed in formal framework of the chaotic growth model that is highly simplified regarding the economic mechanisms represented, but which is extremely detailed from the dynamical point of view. The ratio (a) of agriculture (A) and agri-industry (I) is

$$a = \frac{A}{I} \tag{1}$$

Let I denotes the agri-industry sector (the output sector) , and A the agricultural sector (the input sector).

This particular ratio of input (agriculture) to output (agri-industry) reflects the production technology at a given point in time.
Now, it is assumed that this production coefficient is not constant .

We index a by t, i.e., write a_t to refer to the size at time steps t=0,1,2,3,... Now the growth rate is measured by the quantity already given corresponding to the expression:

$$\frac{a_{t+1} - a_t}{a_t} \tag{2}$$

It is postulated that the growth rate at time t should be proportional to 1 - a_t The growth of the agriculture/agri-industry ratio would change according the following equation, after introducing a suitable parameter γ:

$$\frac{a_{t+1} - a_t}{a_t} = \gamma (1 - a_t) \tag{3}$$

Solving the last equation yields the agriculture/agri-industry ratio growth model, i.e.,

$$a_{t+1} = a_t + \gamma \ a_t (1 – a_t) \tag{4}$$

This model given by equation (4) is called the logistic model. For most choices of γ, there is no explicit solution for (4). Namely, knowing γ and measuring a_0 would not suffice to predict a_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (4) can lead to very interesting dynamic behaviour , such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behaviour of a_t. This difference equation (4) will posses a chaotic region. Two properties of the chaotic solution are important : firstly, given a starting point a_0 the solution is highly sensitive to variations of the parameter γ ; secondly, given the parameter γ , the solution is highly sensitive to variations of the initial point a_0. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

The logistic equation

The logistic map is often cited as an example of how complex, chaotic behaviour can arise from very simple non-linear dynamical equations. The map was popularized in a seminal 1976 paper by the biologist Robert May. The logistic model was originally introduced as a demographic model by Pierre François Verhulst.
It is possible to show that iteration process for the logistic equation:

$$z_{t+1} = \pi z_t (1 - z_t) , \quad \pi \in [0 ,4] \quad , \ z_t \in [0 ,1] \tag{5}$$

is equivalent to the iteration of growth model (4) when we use the identification:

$$z_t = \frac{\gamma}{\gamma+1} a_t \quad \text{and} \quad \pi = \gamma+1 \tag{6}$$

Using (6) and (4) we obtain

$$z_{t+1} = \frac{\gamma}{\gamma+1} a_{t+1} = \frac{\gamma}{\gamma+1} [a_t + \gamma \ a_t (1 - a_t)] =$$

$$= \gamma a_t - \frac{\gamma^2}{\gamma+1} a^2_t$$

Using (5) and (6) we obtain

$$z_{t+1} = \pi z_t (1 - z_t) = (\gamma + 1) \frac{\gamma}{\gamma+1} a_t (1 - \frac{\gamma}{\gamma+1} a_t)$$

$$= \gamma a_t - \frac{\gamma^2}{\gamma+1} a^2_t$$

Thus we have that iterating $a_{t+1} = a_t + \gamma \ a_t (1 - a_t)$ is really the same as iterating $z_{t+1} = \pi z_t (1 - z_t)$ using $z_t = \frac{\gamma}{\gamma+1} a_t$ and $\pi = \gamma+1$. It is important because the dynamic properties of the logistic equation (5) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that :

(i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;

(ii) For $1 < \pi < 3{,}57$ there exist fixed points the number of which depends on π;

(iii) For $1 < \pi < 2$ all solutions monotnically increase to $z = (\pi - 1) / \pi$;

(iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;

(v) For $3 < \pi < 4$ all solutions will continously fluctuate;

(vi) For $3{,}57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

Conclusion

This paper suggests conclusion for use of the agriculture/agri-industry ratio growth model in analyzing the interdependence of the agricultural and agri-industry sector. The model (4) has to rely on specified parameter γ and initial value of the agriculture/agri-industry ratio, a_0. But even slight deviations from the values of parameter γ and initial value of the agriculture/agri-industry ratio, a_0, show the difficulty of predicting a long-term movement of this input coefficient.

A key hypothesis of this work is based on the idea that the coefficient $\pi = \gamma+1$ plays a crucial role in explaining local agriculture/agri-industry ratio stability, where γ is the suitable parameter.

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