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THE CHAOTIC AGRICULTURAL PRODUCTION GROWTH MODEL: THE DANUBE COUNTRIES

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Abstract

The ten Danube countries are : Germany, Austria, Slovakia, Hungary, Croatia, Serbia, Bulgaria, Moldova, Ukraine and Romania. The Danube countries are facing several challenges: environmental threats, insufficient transport and energy connections, uncoordinated economy, education, research. Also, it is important to improve security system in the Danube countries.

The basic aims of this paper are: firstly, to set up a chaotic agricultural production growth model , that is capable of generating stable equilibria, cycles, or chaos, and secondly, to analyze the local stability of agricultural production growth in the Danube countries in the period 1961-2009 .

The estimated model confirms the local stability of agricultural production growth in the Danube countries in the observed period.

Key words: *agricultural production, growth, chaos*

Introduction

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983,1997,), Grandmont (1985), Goodwin (1990), Medio (1993,1996), Lorenz (1993), Shone, R.(1999) , Jablanovic (2010, 2011a, 2011b).

Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (i) extreme sensitivity to initial conditions ; (ii) cause and effect are not proportional; and (iii) nonlinearity.

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The Danube, as Europe's secong logest river, originates in the Black Forest in Germany and flows southeastward for a distance of some 2,872 km, before emptying into the Black Sea. The ten Danube countries are : Germany, Austria, Slovakia, Hungary, Croatia, Serbia, Bulgaria, Moldova, Ukraine and Romania. The Danube countries are facing several challenges: environmental threats, insufficient transport and energy connections, uncoordinated economy, education, research. Also, it is important to improve security system in the Danube countries.

The basic aims of this paper are: firstly, to set up a chaotic agricultural production growth model , that is capable of generating stable equilibria, cycles, or chaos, and secondly, to analyze the stability of agricultural production growth in the Danube countries in the period 1961-2009 (www.fao.org).

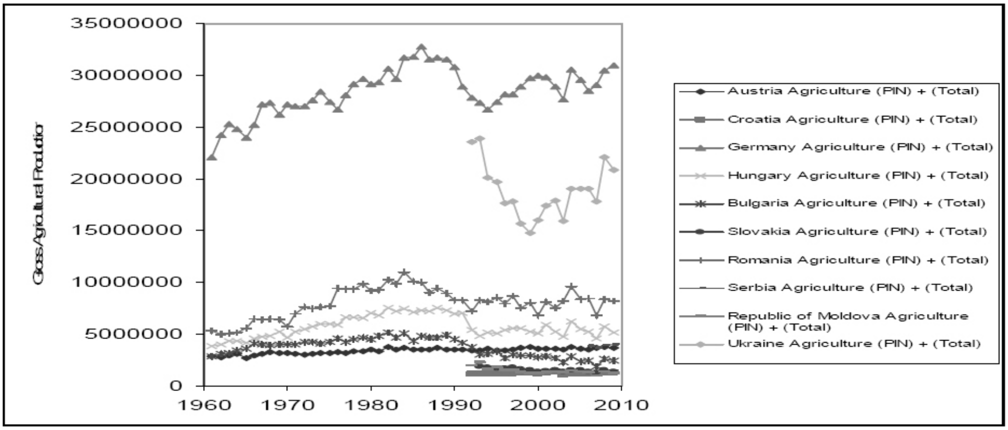

Figure 1. Gross Agricultural Production in the Danube countries (www.fao.org).

A Simple Chaotic Agricultural Production Growth Model

The chaotic agricutlural production growth model is presented by the following equations:

$$\frac{Q_{t+1} - Q_t}{Q_t} = \alpha - \beta \frac{L_t}{Q_t} \tag{1}$$

$$Q_t = \gamma L_t^{1/2} \tag{2}$$

Where Q – agricultural production , L – labour , α, β , γ - coefficients.
Equation (1) describes the relation between the rate of agricultural production growth and agricultural productivity; relation (2) determines production function.
Further, by substitution one derives:

$$Q_{t+1} = (1+\alpha) Q_t - \frac{\beta}{\gamma^2} Q_t^2 \tag{3}$$

Further, it is assumed that the current value of the agricultural production is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the agricultural production growth rate depends on the current size of the agricultural production, Q , relative to its maximal size in its time series Q^m. We introduce q as $q = q/q^m$. Thus q range between 0 and 1. Again we index q by t, i.e., write q_t to refer to the size at time steps t = 0,1,2,3,... Now growth rate of the agricultural production is measured as

$$q_{t+1} = (1+\alpha)\, q_t - \frac{\beta}{\gamma^2}\, q_t^{\ 2} \qquad (4)$$

This model given by equation (4) is called the logistic model. For most choices of α , β , and γ there is no explicit solution for (4). This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

Logistic Equation

The logistic map is often cited as an example of how complex, chaotic behaviour can arise from very simple non-linear dynamical equations. The map was popularized in a seminal 1976 paper by the biologist Robert May. The logistic model was originally introduced as a demographic model by Pierre François Verhulst.

$$z_{t+1} = \pi z_t (1 - z_t), \quad \pi \in [0,4], \quad z_t \in [0,1] \qquad (5)$$

$z_t = \frac{\beta}{(1+\alpha)\gamma^2} q_t$ is equivalent to the iteration of growth model (4) when we use the identification

and $\pi = (1+\alpha)$ (6)

Using (4) and (6) we obtain:

$$z_{t+1} = \frac{\beta}{(1+\alpha)\gamma^2} q_{t+1} = \frac{\beta}{(1+\alpha)\gamma^2}\left[(1+\alpha)\, q_t - \frac{\beta}{\gamma^2} q_t^{\ 2}\right] =$$

$$= \left(\frac{\beta}{\gamma^2}\right) q_t - \left[\frac{\beta^2}{\gamma^4 (1+\alpha)}\right] q_t^{\ 2}$$

On the other hand, using (5) and (6) we obtain:

$$z_{t+1} = \pi z_t (1 - z_t) = (1+\alpha)\left[\frac{\beta}{(1+\alpha)\gamma^2}\right] q_t \left[1-\left(\frac{\beta}{(1+\alpha)\gamma^2}\right) q_t\right]$$

$$\left(\frac{\beta}{\gamma^2}\right) q_t - \left[\frac{\beta^2}{\gamma^4 (1+\alpha)}\right] q_t^{\ 2} \quad 3$$

Thus we have that iterating (4) is really the same as iterating the logistic equation (5). It is important because the dynamic properties of the logistic equation (10) have been widely analyzed (Li and Yorke (1975), May (1976)).
It is obtained that :(i) For parameter values $0 < \pi < 1$ all solutions will converge to z = 0; (ii) For $1 < \pi < 3{,}57$ there exist fixed points the number of which depends on π; (iii) For $1 < \pi < 2$ all solutions monotnically increase to $z = (\pi - 1) / \pi$; (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$; (v) For $3 < \pi < 4$ all solutions will continously fluctuate; (vi) For $3{,}57 < \pi < 4$ the solution become "chaotic" wihch means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

Empirical Evidence

The main aim of this paper is to analyze the agriculture production growth stability in the period 1961-2009 , in the Danube countries , by using the presented non-linear, logistic agriculture production growth model (4) or ,

$$q_{t+1} = \varpi q_t - \vartheta q_t^{\ 2} \qquad (7)$$

where q – agriculture production, $\omega = \pi = (1+ \alpha)$ and □ = β / γ^2
Firstly, we transform data on the agricultural production (www.fao.org) from 0 to 1, according to our supposition that actual value of the agricultural productin, Q, is restricted by its highest value in the time-series, Q^m. Further, we obtain time-series of $q = Q / Q^m$. Now, we estimate the model (7) . The results are presented in Table 1. :

Table 1. The estimated model (7)

Austria (1961-2009)	R=.86579 Variance explained: 74.958%		
		ω	υ
	Estimate	1.19092	.206236
	Std.Err.	.07705	.084823
	t(46)	15.45743	2.431372
	p-level	.00000	.018996

Croatia (1992-2009)	R=.21658 Variance explained: 4.6908% ω υ Estimate 1.9208781 .034755 Std.Err. .250618 .281934 t(15) 7.664576 3.670200 p-level .000001 .002274
Germany (1992-2009)	R=.88391 Variance explained: 78.130% ω υ Estimate 1.19844 .220275 Std.Err. .06638 .075490 t(47) 18.05392 2.917948 p-level .00000 .005390
Hungary (1992-2009)	R=.87567 Variance explained: 76.680% ω υ Estimate 1.14971 .184237 Std.Err. .07444 .090159 t(46) 15.44382 2.043465 p-level .00000 .046758
Bulgaria (1992-2009)	R=.91645 Variance explained: 83.988% ω υ Estimate 1.07982 .108139 Std.Err. .07379 .091836 t(46) 14.63445 1.177520 p-level .00000 .245044
Slovakia (1992-2009)	R=.68093 Variance explained: 46.367% ω υ Estimate 1.257074 .320348 Std.Err. .189525 .216456 t(14) 6.632775 1.479969 p-level .000011 .161031
Romania (1992-2009)	R=.85897 Variance explained: 73.784% ω υ Estimate 1.22130 .283672 Std.Err. .08550 .108980 t(46) 14.28388 2.602958 p-level .00000 .012398
Serbia (2006-2009)	R=.20068 Variance explained: 4.0273% ω υ Estimate 2.318246 1.364613 Std.Err. 1.211033 1.259764 t(1) 1.914272 1.083230 p-level .306469 .474579

Republic of Moldova (1992-2009)	R=.62113 Variance explained: 38.581%		
		ω	υ
	Estimate	1.300287	.490319
	Std.Err.	.176036	.245817
	t(15)	7.386469	1.994653
	p-level	.000002	.064581
Ukraine (1992-2009)	R=.68794 Variance explained: 47.327%		
		ω	υ
	Estimate	1.323263	.416735
	Std.Err.	.167000	.203719
	t(15)	7.923716	2.045641
	p-level	.000001	.058744

Conclusion

This paper suggests conclusion for the use of the chaotic agricultural production growth model in predicting the fluctuations of the agricultural production. The model (4) has to rely on specified parameters α , β , and γ initial value of the agricultural production, q_0. But even slight deviations from the values of parameter γ and initial value of agricultural production, q_0, show the difficulty of predicting a long-term agricultural production movement.

A key hypothesis of this work is based on the idea that the coefficient $\pi = (1+ \alpha)$ plays a crucial role in explaining local stability of the agricultural production growth, where, α as a coefficient presents the relation between agricultural growth rate and agricultural productivity.

The estimated model confirms the local stability of agricultural production growth in the Danube countries in the observed period.

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HAOTIČAN MODEL RASTA POLJOPRIVREDNE PROIZVODNJE: PODUNAVSKE ZEMLJE

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Rezime

Deset podunavskih zemalja su: Nemačka, Austrija, Slovačka, Madjarska, Hrvatska, Srbija, Bugarska, Moldavija, Ukrajina i Rumunija. Podunavske zemlje se suočavaju sa nekoliko izazova: pretnje problemima životne sredine, nedovoljne saobraćajne i energetske povezanosti, nekoordinisanim privredama, obrazovanjem, naučenim istraživanjem.Takodje, značajno je poboljšati bezbednosni sistem u podunavskim zemljama.

Osnovni ciljevi ovog rada su : prvo, postaviti haotični model rasta poljoprivredne proizvodnje, koji je u mogućnosti da generiše stabilnu ravnotežu, cikluse ili haos, i drugo, da analizira lokalnu stabilnost rasta poljoprivredne proizvodnje upodunavskim zemljama u periodu 1961-2009 (www.fao.org).
Ocenjeni model potvrdjuje egzistenciju lokalne stabilnosti rasta poljoprivredne proizvodnje u podunavskim zemljama u posmatranom periodu.

Ključne reči: *poljoprivredna proizvodnja, rasta, haos.*

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