MODIFICATION OF THE COBWEB MODEL INTO GENERALIZED LOGISTIC EQUATION FOR THE WHEAT PRICE ANALYSIS

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A R T I C L E I N F O

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A B S T R A C T

In the paper we constructed the new wheat growth model, based on the generalized logistic equation. Starting from the theoretical framework of the cobweb model, we adapted generalized logistic equation to better fit the real data of wheat prices, according to the presented wheat growth model. The aim of the paper is to present how logistic and generalized logistic equations can be used for both prediction of wheat prices and for the wheat price stability analysis. Data analysis showed better performances of the generalized logistic map in comparation with the conventional logistic map as a main result of this paper. For estimated parameters of the model the bifurcation diagrams also have been presented to show stability of wheat price over time. The conclusion is that the proposed model can be useful in predicting future wheat prices in the short-run period, as well as for the analysis of stability in conditions of uncertainty, which is also a recommendation for the application of the model in the future research.

Introduction

Since 1930 until now iterated maps were considered very important in the modelling and processing of many science fields. One of the most famous maps comes from the so called continuous logistic equation which was introduced by Verhulst in the middle of the 19th century. The dynamical behaviour of this continuous equation is

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trivial compared with that one presented by the discrete logistic map introduced in the 1960s (e.g., Ausloos and Miskiewicz, 2006; Lopez-Ruiz and Fournier-Prunaret, 2004; Stanojević and Kukić, 2018). The use of logistic equation for price stability analysis can be related with the famous cobweb model. The prices of non-storable goods are observed daily and are subject to strong fluctuations, while production of such goods requires a longer time. The cobweb model, originally presented by Kaldor in 1934, has been extensively investigated in a discrete time deterministic context. The model describes fluctuations of equilibrium prices as an independent market for non-storable good that takes one time to produce, so that producers must form price expectations one period ahead. Applications of the cobweb model mainly concern agricultural markets, such as the classical examples of cycles in wheat or corn prices (Goeree and Hommes, 2000). Within the early cobweb model producers simply form naïve expectations and demand and supply schedules are linear. It is important to explain the reasons why prices of goods in the agricultural sector fluctuate over time. According to the cobweb theorem, assuming that the supply and demand functions are linear, it can be shown that if the slope of the supply function is greater than the slope of the demand function, the oscillations around the equilibrium price and equilibrium quantity will decrease over time. If the opposite case holds, so that the slope of the demand function is greater than the slope of the supply function, then the oscillations will intensify, and the market will move away from equilibrium. This type of analysis within the cobweb theorem is particularly important in the field of agrarian economy, since agricultural production is known for the fact that supply oscillations often occur - due to the great influence of external factors, such as weather conditions, so the analysis of equilibrium stability is also important. According to the cobweb model, producers operate in market where production must be chosen before prices are observed, so their choices depend on prices they expect to prevail at harvest time. By assuming that producers take the current price as an estimate of the expected price, stability of the market equilibrium is shown to depend on relative elasticity of demand and supply functions. The traditional cobweb model therefore represents a very useful tool for the analysis of price dynamics (Milovanovic, 2011). One example of the agricultural production market that deserves the attention of analysts is certainly the wheat market. It has been shown in the literature that in the case of the wheat market there is a constant oscillation around the equilibrium price, so that a certain cycle can be observed in which the price of wheat alternately jumps and falls, depending on whether the supply is insufficient or excessive in relation to the demand. As a result of constant fluctuations in the price of wheat, the profitability of this agricultural activity is variable, which then points to the need for state intervention to stabilize the price of wheat with appropriate state measures and thereby prevent permanent fluctuations in the offer on the wheat market (Dieci and Westerhoff, 2009).

However, the traditional setting of the cobweb model, which assumes that the supply and demand functions are linear, has been modified over time, so a significant number of works have been published recently that start from different settings (Mitra and

Boussard, 2008; Evans and McGough, 2020). Also, the model has only theoretical value, and the possible range of dynamic outcomes is basically restricted to either dampened or exploding oscillations around the equilibrium price. As a result, the interest of researchers in creating new models that would enable the analysis of stability and price prediction of agricultural products, which would more realistically correspond to the observed characteristics of the market, is expanding. In the last twenty years, the growing popularity of nonlinear dynamics in economic analysis has brought about a renewed interest in cobweb models, and the basic setup has been modified in a way which include nonlinear elements. Some authors (e.g., Chiarella, 1988; Day, 1994; Hommes, 1998) considered nonlinear demand and supply functions with different adaptive expectations schemes.

The aim of the paper is to present how logistic and generalized logistic equations can be applied for the price stability analysis starting from the cobweb model – and eventually for making predictions about the future wheat price. To show the implementation of dynamic stability analysis in terms of difference equations we analyse the wheat prices. In general, the prices of agricultural products are very volatile during time, so it is important to examine the price stability for these products. This is valuable in recent period, as we are witnessing dramatic upheavals in the agricultural market due to different disruptions, e.g., Covid-19 pandemic or Russia-Ukraine war. We develop the new wheat growth model based on generalized logistic equation and test the power of the model based on real data of wheat prices. It should be emphasized that the new model may be valuable for researchers as it keeps mathematical elegancy and also provides empirical accuracy. For that reason the model could be used as a replacement of the classical logistic equation in many applications, not only in economics. In the paper, it will be shown that the new growth model can be used for wheat price predictions in terms of stability, which may be interesting for different agricultural analysts. Considering the importance of wheat on a global level as a basic commodity, it is useful to create an appropriate methodological framework for the analysis of wheat price movements, especially due to the volatility of wheat prices on the world market.

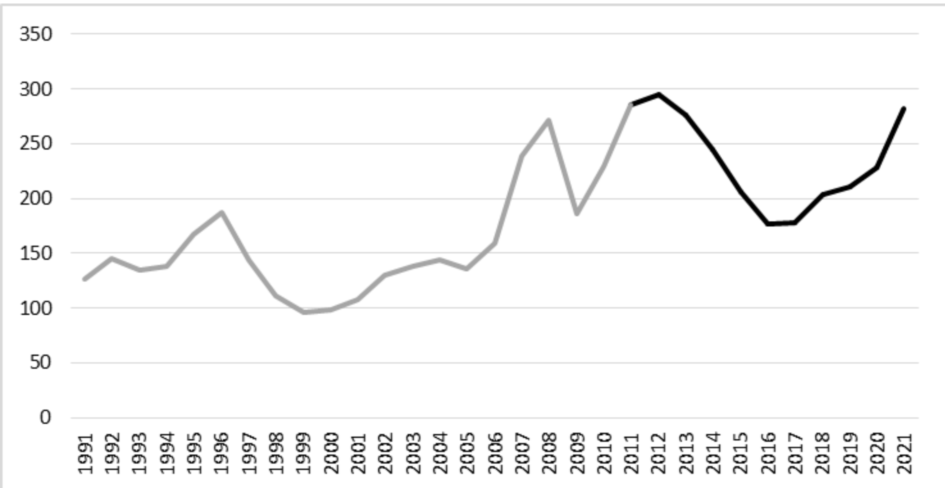
Materials and methods

Wheat market characteristics within the framework of the price stability-Volatility of wheat prices

According to a recent FAO report (2022), it is expected that in the coming period, wheat price oscillations will be especially pronounced if we consider the decline in production with growing demand globally, which will cause the price of wheat to rise globally due to a lack of supply. As stated in the report, global wheat production in 2021 compared to 2020 decreased by nearly 1%, primarily due to the decline in wheat production in Russia, Canada, and the USA. However, wheat consumption is about 2% higher in 2021 compared to the previous year. The global increase in demand for wheat, as the report points out, reflects both the growth of the world population and the increase in demand for food - especially in the EU, China, the UK, and the USA. As a result of these developments, the price of wheat on the world market increased from $227/mt to $281/mt, which represents a significant increase of about 25%. It is emphasized that the special attention of the states in the next period should be directed towards the adoption of measures that should mitigate the negative consequences of this trend in the price of wheat on the market, which are the result of disruptions caused by both the previous covid pandemic and the current crisis surrounding the Russian-Ukrainian conflict.

However, such frequent changes in the price of wheat are not typical. As can be seen from the Figure 1, historically, in the period from 1991 to 2021, significant fluctuations in the price of wheat can be observed on the world market. Until 2006, the price of wheat was relatively stable around the value of $150/mt - except for 1999 and 2000, when the price was around $100/mt. After 2006, as can be seen from the Figure, there are significant fluctuations in the price of wheat on the world market. Thus, the price of wheat decreased from $270/mt in 2008 to $185/mt, so the drop was as much as 30%. Immediately after that, there was a significant increase in the price of wheat at the global level from $185/mt in 2009 to $285/mt in 2010, so the price of wheat almost doubled in such a short period. Such pronounced price changes in this case are primarily a reflection of the disturbances caused by the global financial crisis in 2008. More significant changes in the price of wheat can be observed in the recent period, especially in the last ten years. Until 2016, a downward trend in the price of wheat was noticeable, followed by a period in which an increase in the price of wheat was recorded. This growing trend has been particularly pronounced in the last few years, as already pointed out in the FAO report.

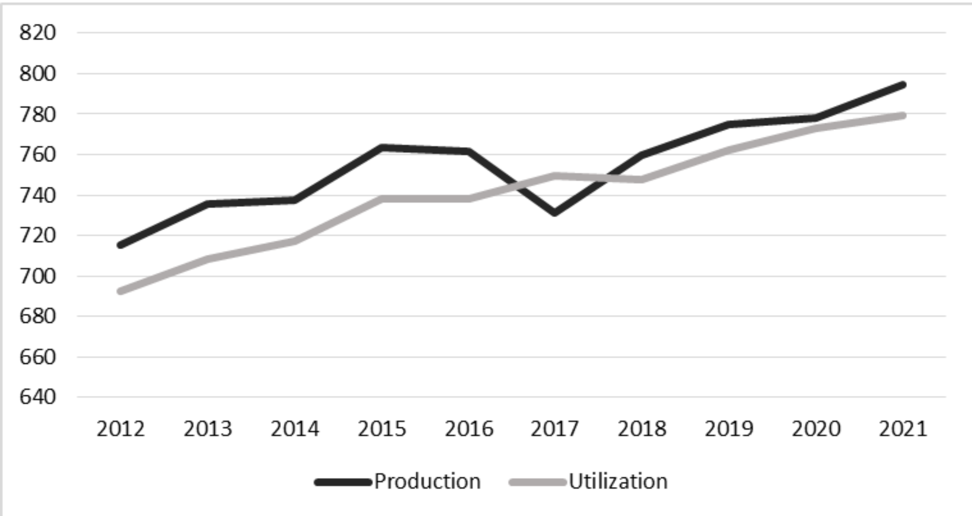
Figure 1. Wheat price trend at the world level, 1991-2021 (in $/mt)



Source: FAO (https://www.fao.org/faostat/en/#data)

Such recorded volatility of wheat prices is not surprising but reflects the characteristics of the wheat market. As mentioned, demand is relatively stable, while supply is a subject to frequent changes due to the influence of external factors over which agricultural producers do not have influence. This situation is confirmed by the Figure 2, which shows the trends in wheat production and consumption in the period from 2012 to 2021. In this observed period of ten years, it can be noted that wheat production was higher than consumption during the entire observed period, except for 2017. Only in 2017, due to a poor agricultural season, wheat production was lower than consumption, which was reflected in the price of wheat. As can be seen from the Figure 1, until 2017, when the supply was greater than the demand, the price of wheat fell, after which there was a change in the trend, so that after 2017, the price of wheat gradually increased. This is quite in line with expectations, because if supply is greater than demand there is pressure for the price to fall, and if demand is greater than supply there is pressure for the price to rise.

Figure 2. Wheat production and utilization trends at the world level, 2012-2021 (in mill. tones)



Source: FAO (https://www.fao.org/faostat/en/#data)

Methods of logistic and generalized logistic map, with stability analysis

Although discrete logistic map was popularized in the 1970s by Robert May in his well-known paper published in the journal *Nature* (May, 1972), another complex map based on the iterated empirical reproduction curves of fish was introduced earlier by William Ricker in 1954 (Ricker, 1954). Furthermore, the analysis of many iterated maps was studied such as generating random numbers from the logistic map by John von Neumann in 1940 (Radwan, 2013).

Akhmet et al., 2014, state that irregularity is an inherent feature of reality and that regularity, as reflected in a constant solution of a model or a periodic and even almost periodic motion in mathematical sense, is a good assumption in natural science applications, but less so in social science. This, as they say, was pointed out in early scientific work and has been widely discussed in recent years (e.g., Rosser, 2000). One way of introducing irregularity in economics is by allowing for stochastic processes. A different approach is based on generating chaos in deterministic differential equations. The main property of a second approach is sensitivity, which can be interpreted as unpredictability in real world problems. In the literature, this is known as the butterfly effect (Lorenz, 1963). Differential equations have been used to model the dynamical characteristics of population system in the case of species with overlapping generations and by difference equations in the case of species with non-overlapping generations. Especially, when we cannot solve differential equation explicitly, we propose a finite difference scheme to solve result numerically. In the situation when population growth is not continuous but seasonal with overlapping generations, difference equations are also well models. The analysis of a nonlinear difference equation has been the high attractive topic in mathematics and many other fields in the resent years. In fact, the main problem is to find the closed-form solutions of the nonlinear difference equation, which is very challenging task. Today, there are many methods to transform nonlinear difference equations into linear form which solution is already known. With this transformation into linear form a large class of nonlinear difference equations were resolved in closed-forms (see El-Moneam and Alamoudy, 2014; Elsayed, 2014; Elsayed and Ahmed, 2016). Qualitative studies of a higher order rational difference equations and global asymptotic stability are also a topic of many resent papers (see Halim, 2015; Ibrahim, 2014).

The logistic map is a non-linear recurrence relation with a single control parameter r and in its first showing by Pierre Verhulst it described the population size x relative to the generation n as follows

$$x_{n+1} = f(x_n) = rx_n(1 - x_n) \tag{1}$$

and it is one of the simplest forms of a chaotic process (Li and Yorke, 2004). The parameter r is fixed and if we consider mapping as a function of r (since $f: [0,1] \to [0,1]$, then $r < 4$) it is well-known that chaos appears for some values of the parameter. The parameter r represents the growth rate of the population. After many iterations of x it reaches some values which are independent of its starting value for some values of the parameter. Some characteristic intervals for r that define different type of function behavior are given in the Appendix A.

Generalization of the logistic map (1) is introduced in the literature and has the following form

$$x_{n+1} = rx_n^p(1 - x_n^q),\ x_n \in [0,1],\ p > 0,\ q > 0. \quad (2)$$

Analysis of (2) for parameters $p = 1, q = 2$ is done in Rak and Rak, 2015, and more advance analysis of the equation (2) for cases $(p,q) = (\alpha,\alpha), (p,q) = (1,\alpha), (p,q) = (\alpha,1)$ is presented in Radwan, 2013. Since the model we propose in this paper is in the form (2), with $(p,q) = (1,\alpha)$, we will briefly present stability analysis for that case.

Denote left side of (2) as a function:

$$f(x,r,\alpha) = rx(1 - x^\alpha). \quad (3)$$

To achieve that $f: [0,1] \to [0,1]$ and also $f^n: [0,1] \to [0,1]$ for any iteration, the maximal value of the function must remain less than or equal to 1, so with this request we get first constrain for parameters' values. Consider (3) as a function of x, and denote $f_{r,\alpha}(x) = rx(1 - x^\alpha)$. The maximum of the function is reached for

$$x_{max} = \left(\frac{1}{1+\alpha}\right)^{\frac{1}{\alpha}}. \quad (4)$$

From the demand that $f_{r,\alpha}(x_{max}) \leq 1$ we obtain that for fixed value of parameter α, the value of parameter r depends on α and the range for r is $\left[0, \frac{(\alpha+1)^{1+\frac{1}{\alpha}}}{\alpha}\right]$. As a next step, we look for a fixed points with respect to r. Starting from $f(x^*,r,\alpha) = x^*$, similar as in conventional logistic map, the fixed points are $x_1^* = 0$ and $x_2^* = \left(1 - \frac{1}{r}\right)^{\frac{1}{\alpha}}$, for $r > 1$. To consider stability we must investigate first derivative in the fixed points: $f_r'(x) = r(1 - (\alpha + 1)x^\alpha)$. When we calculate values of the derivative in fixed points, we obtain $f_r'(x_1^*) = r$ and

$f_r'(x_2^*) = \alpha(1 - r) + 1$, so we conclude that for $0 < r < 1$ the only fixed point is x_1^* and it is stable. For $r > 1$, x_1^* becomes unstable and x_2^* is stable for $|\alpha(1 - r) + 1| < 1$, which is for $r \in \left(1, \frac{\alpha+2}{\alpha}\right)$.

Results and Discussions

A producer wheat price growth model based on logistic map

The simple model is based on supply-demand model and the change of the price as a function of surplus of demand for wheat. It is well known that the simple supply-demand model is one of the typical examples of the use of dynamical systems in economics. The simplest linear supply-demand model with the naive price expectation in terms of stability does not show chaotic motion of prices. The use of adaptive price expectation instead of naive can show much more complicated dynamics, as presented in Hommes, 1991. In the Appendix A we presented overview of linear supply-demand model, which can be transformed into logistic equation. This linear supply-demand model gives equation: $p_{t+1} = \alpha p_t - \beta p_t^2$, where we denoted $p_t = P_t / P_{max}$ and P_t is a wheat price in the period t, P_{max} is maximal wheat price during the considered period, and the model may be transformed into logistic form ($z_{n+1} = f(z_n) = rz_n(1 - z_n)$), with appropriate change of variables. The dynamical analysis of the previous model is already well known in the literature, and we gave it in the Appendix A.

A producer wheat price growth model based on generalized logistic map

In this subsection we developed a new model based on nonlinear supply and demand functions. We choose the shape of the functions to obtain a model of the price growth which is reduced to generalized logistic equation ($z_{n+1} = f(z_n) = rz_n\left(1 - z_n^q\right)$). The model which we presented in the paper has arbitrary power q, which can be chosen to best fit the observed data and in this respect, this added parameter increase the flexibility of the system. Since the scientists' goals when they make a new model are mathematical elegance and empirical accuracy, we considered that the model we presented keeps the elegance of the logistic map while additional parameter increases empirical accuracy. In the Appendix B we presented transformation, step by step, of our model into generalized logistic form. In this subsection we gave only the main result.

Instead of linear supply-demand model we replaced appropriate equations by following:

$$D_t = a - bP_t^q, S_t = -c - dP_t^q$$

where $q > 0$ is arbitrary parameter, $a, b > 0$ and $c, d < 0$. In order to obtain chaotic wheat producer price growth model, we suppose that relative change in wheat price is proportional with surplus of demand for wheat: $\frac{P_{t+1} - P_t}{P_t} = \theta(D_t - S_t)$. Further, we obtain:

$$P_{t+1} = P_t\left(1 + \theta(a + c) - \theta(b - d)P_t^q\right)$$

where a, b, c, d, θ and q are appropriate parameters. Again, in order to norm the price, we divided previous equation by P_{max}, denoted $p_t = P_t/P_{max}$, and obtained second model:

$$p_{t+1} = p_t\left(1 + \theta(a + c) - \theta P_{max}^q (b - d) p_t^q\right) = \alpha p_t - \beta p_t^\gamma .$$

Finally, with change of variable presented in the Appendix B ($z_t = \left(\frac{\theta P_{max}^q (b-d)}{1+\theta(a+c)}\right)^{1/q} p_t$) , previous relation is equivalent to

$$z_{t+1} = \left(1 + \theta(a + c)\right) z_t\left(1 - z_t^q\right) = r z_t(1 - z_t^q)$$

what is in the form of generalized logistic map. Further, it is possible to give stability analysis with the respect of the parameter $r = 1 + \theta(a + c)$ (more precisely, of parameters a, c, θ), what is beyond the scope of this paper. It should be bear in mind that with the introduction of the appropriate changes of variables, which leads to the generalized logistic equation, the previous interpretation of the corresponding parameters of the model (for which the stability analysis is performed) is lost – as new variables have been introduced. This is precisely one of the limitations of the presented model, which should be deal with in a broad analysis in the further research.

To test the power of developed model, we used data about wheat prices for Russia, China, Turkey, Australia, Serbia, and Brazil, during the period 1991-2021. Russia, China, Turkey, Australia, and Brazil were selected as the largest producing countries of wheat. However, in order to test sensitivity of the models, Serbia was also selected as a small producing country of wheat. Prices are in US$/tone and available on www.fao.org.

First model is:

$$p_{t+1} = \alpha p_t - \beta p_t^2$$

and further may be transformed into logistic map. We have to estimate parameters α and β in this model.

We also analysed the wheat producer price growth with generalized logistic map, given with the suggested second model:

$$p_{t+1} = \alpha p_t - \beta p_t^\gamma, \gamma > 1.$$

To select the better model, it should be noted that the general framework for R^2 does not work out correctly if the regression model is not linear. Spiess and Neumeyer, 2010, investigated the effect of using R^2 to assess the goodness-of-fit for models that are not linear. Their study ran thousands of simulations and found that R^2 leads to false conclusions about which nonlinear models are better. Further, they explained that computing R^2 for nonlinear model indicates the following problems: R^2 is consistently high for both excellent and appalling models; R^2 will not rise for better models all of the time; using of R^2 in order to pick up the better model leads to the proper model only 28-43% of the time. There are other goodness-of-fit measures which can be used for regression models that are not linear. For instance, it is common to use the standard error of regression and confidence interval. Smaller values of the standard errors and narrower confidence intervals indicate the better model.

In this part of the subsection, we provided estimates of all parameters for both models, α, β and γ, standard errors and 95% confidence intervals of those parameters, and for all considered countries the results are given in the Table 1.

Results suggested that generalized logistic model provides better estimates of the parameters. More precisely, standard errors are smaller and confidence intervals are narrower compared to the conventional logistic model. For this type of data nonlinear regression is much more flexible in the shapes of the curves that it can't fit. The previous model considers adaptive adjustments on the quantity of wheat produced instead of adaptive expectations on prices. From an economic point of view, chaos occurs more likely the faster producers adjust production and the more inelastic the market demand is.

Table 1. Parameter estimations, standard errors and 95% confidence intervals of the two models and six selected countries

	Param.	**Estim.**	**Std.**	**95% Confidence interval**	
			Error	**Lower Bound**	**Upper Bound**
Russia					
First model:	α	1.376	0.165	1.038	1.714
	β	0.603	0.221	0.148	1.058
Second model:	α	1.044	0.067	0.906	1.181
	β	0.431	0.145	0.133	0.729
	γ	9.942	6.385	-3.209	23.092
China					
First model:	α	1.156	0.076	1	1.312
	β	0.163	0.096	-0.035	0.362
Second model:	α	1.078	0.028	1.02	1.137
	β	0.196	0.058	0.076	0.315
	γ	14.865	9.527	-4.831	34.562
Turkey					
First model:	α	1.211	0.115	0.976	1.446
	β	0.329	0.165	-0.01	0.668
Second model:	α	1.03	0.044	0.94	1.12

	Param.	Estim.	Std.	95% Confidence interval	
			Error	Lower Bound	Upper Bound
	β	0.283	0.099	0.078	0.487
	γ	11.015	10.695	-10.969	32.998
Australia					
First model:	α	1.411	0.141	1.121	1.7
	β	0.612	0.2	0.202	1.021
Second model:	α	1.107	0.079	0.945	1.269
	β	0.418	0.12	0.171	0.665
	γ	6.068	3.72	-1.579	13.716
Serbia					
First model:	α	1.829	0.26	1.263	2.394
	β	1.138	0.338	0.402	1.873
Second model:	α	1.141	0.096	0.93	1.352
	β	0.593	0.158	0.246	0.94
	γ	8.719	5.212	-2.752	20.19
Brazil					
First model:	α	1.322	0.136	1.04	1.604
	β	0.451	0.189	0.058	0.844
Second model:	α	1.083	0.058	0.963	1.203
	β	0.327	0.113	0.09	0.563
	γ	8.426	5.683	-3.427	20.28

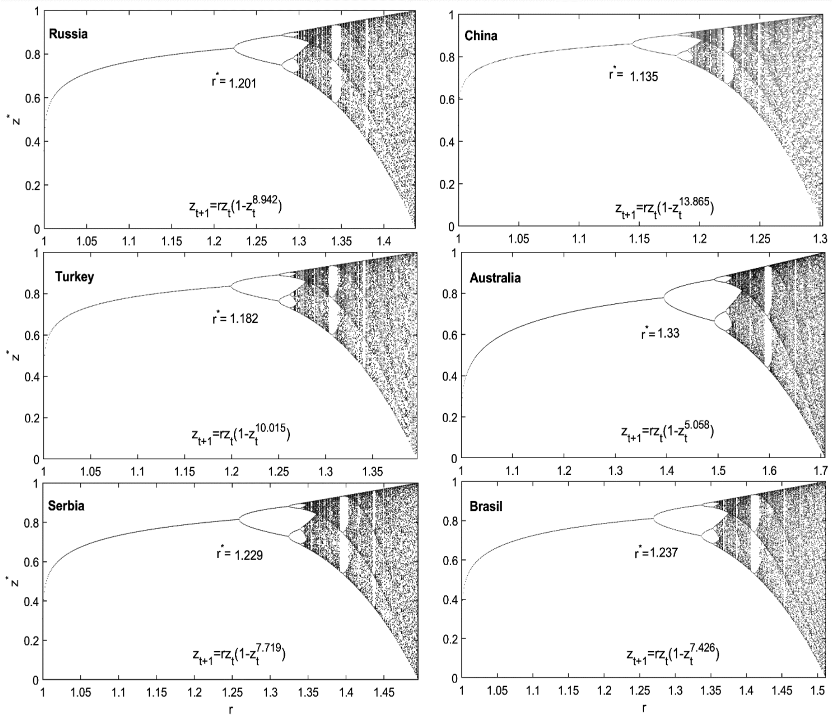
Source: Authors' calculations

Further, for each obtained value of γ we plotted bifurcation diagrams of generalized logistic map

$$z_{t+1} = rz_t(1 - z_t^{\gamma-1}).$$

From corresponding bifurcation diagrams (Figure 1) it may be concluded that for values of the coefficients from second model for each data set, the prices will converge toward stable steady state, with appropriate value of parameter r.

Figure 1. The dynamics of the second model. The three lines of panels present bifurcation diagrams for the wheat price with respect to the parameter r, for the six countries: Russia, China, Turkey, Australia, Serbia and Brazil



Source: Authors' calculations

A bifurcation diagram is a powerful tool for researching how the dynamical behaviour of a nonlinear model depends on a single parameter. As a model parameter changes, a bifurcation diagram is a qualitative change. For instance, a fixed point becomes unstable if one or more eigenvalues of the linearized dynamics around the fixed point cross the unit circle.

Figure 1 presents bifurcation diagrams for parameter r. The bifurcation diagram plots only stable equilibrium points and shows the long-term behaviour for all values of r at once. Bifurcation in the Figure 1 shows that for example, in the case of Russia, the wheat price is stable for $r \in (1{,}1.201)$. With the increase in r, the first bifurcation occurs at r =1.201 and the second between r =1.25 and r =1.3, the third around r =1.3, ..., and then chaos occurs. Generally, a bifurcation occurs at a parameter value $r = r_0$ if the global dynamical behaviour of the function f_r undergoes some qualitative change as r passes through r_0. There are number of different types of bifurcations that can occur, depending on the nature of the qualitative behaviour under consideration.

The bifurcations that are evident in the Figure 1, in the case of Russia, for the range $1 < r < 1.3$ are called *period doubling bifurcations*.

The bifurcation values for these six countries are $r = 1.201$, $r = 1.135$, $r = 1.182$, $r = 1.33$, $r = 1.229$, $r =1.237$. It is therefore more useful to think of this point (bifurcation value for every country separately) as a period-doubling bifurcation. At that point a stable period-two orbit is born out. When r increases each of these two points bifurcates into two new points, as can be seen from the figure. These four points together constitute a period-four solution of the map. As r moves through a sequence of higher values, an infinite series of bifurcations is created by such period-doubling. It is easily illustrated that we move from stability through a sequence of a period doubling bifurcations to chaos.

Notice in figure the 'windows' occurring. Three are marked on the diagram. Such windows represent stable periodic orbits that are surrounded by chaotic behaviour (the dark regions).

Conclusions

In the paper we implemented generalized logistic equation in the chaotic wheat producer price growth model. Based on data analysis we showed better performances of the model compared to the conventional logistic map. The impact of estimated parameters in both models related to the wheat producer price has a sign in the expected direction. From the review of the generalized logistic map stability, we conclude that model predicts stable growth of the price, what is in the correlation with world's trend of the foods' prices growth, for the observed period. The analysis was performed using the data regards wheat prices in the period 1991-2021, after which the Ukrainian crisis followed, and therefore a big jump in wheat prices on the market occurred. In general, since it is difficult to predict prices even for a shorter period, we considered that the proposed model could more adequately predict price movements in the years to come. The mathematical apparatus used in this paper is well-known in the literature, but not with application in the field of market analysis of agricultural products, as far as the authors are aware. The paper also provides the theoretical frame of equilibrium point stability analysis, which represents a good basis for the further research.

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Conflict of interests

The authors declare no conflict of interest.

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Appendix. Supplementary material

The following is the Supplementary material related to this article.

Appendix A

Suppose to have a good with the price P_t in the period *t*. The simplest dynamical system can be obtained with assumption that supply is function of expected price. The naive price expectation means that $P_t^e = P_{t-1}$ and $S_t = -c - dP_t^e, c, d < 0$, and by equating supply and demand we obtain simple discrete time cobweb model with well known stability of price, that can be stable, unstable or oscillate between two values in two-cycle. More complicated dynamics can be achieved either with use of adaptive price expectations or with nonlinear functions for demand or supply. The most often, as nonlinear supply function is chosen the S-shaped function $S_t = arctg(\lambda \cdot (P_{t-1} - 1))$

As the demand function is decreasing function of the price, while the supply function is increasing function of the price, let us start from the linear functions:

$$D_t = a - bP_t \tag{5}$$

$$S_t = -c - dP_t \tag{6}$$

where $a, b > 0$ and $c, d < 0$. One can easily find the equilibrium price by equating supply and demand.

In order to obtain chaotic wheat producer price growth model, beside linear supply and demand functions (5) and (6), we suppose that relative change in wheat price is proportional with the surplus of demand for wheat:

$$\frac{P_{t+1} - P_t}{P_t} = \theta \cdot (D_t - S_t) \tag{7}$$

Substituting (5) and (6) into (7), we obtain:

$$P_{t+1} = P_t(1 + \theta(a + c) - \theta(b - d)P_t). \tag{8}$$

Relation (8) is in similar form as conventional logistic map:

$$x_{n+1} = f(x_n) = rx_n(1 - x_n) \tag{9}$$

what we will prove below.

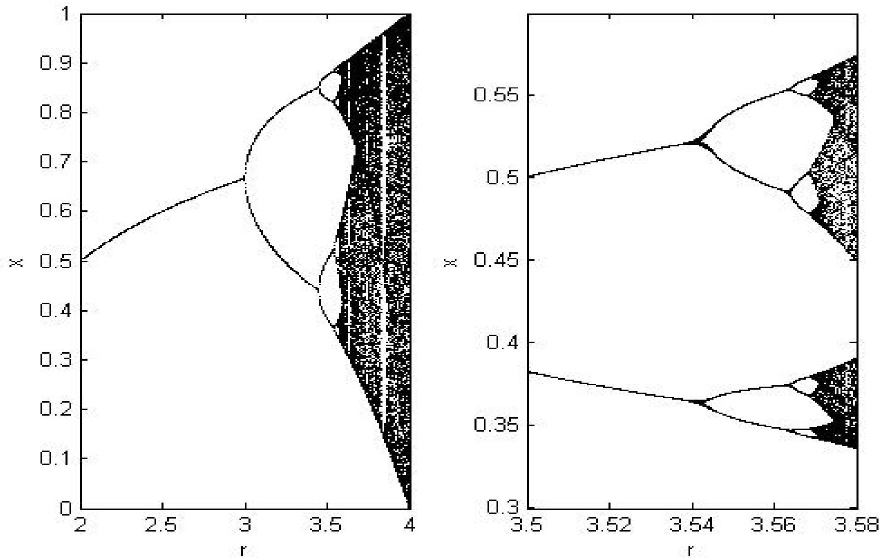
Some characteristic intervals for r that define different type of function behavior (and gives as stability analysis) are:

- $0 < r < 1$: the only stable fixed point is ${x_1}^* = 0$
- $r = 1$: in this case we have exchange of stability between two points, or transcritical bifurcation
- $1 < r < 2$: the fixed point ${x_1}^*$ become unstable, but other fixed point $x_2^* = 1 - \frac{1}{r}$ becomes stable, and all solutions monotonically increase to x_2^*
- $2 < r < 3$: x_2^* is still stable fixed point, but solutions oscillatory converge towards it
- $r = 3$: flip bifurcation for x_2^* (see Figure 2)
- $3 < r \leq 1 + \sqrt{6}$: x_2^* becomes unstable and period 2 cycle shows up which is asymptotically stable
- $1 + \sqrt{6} < r \leq 3.544$: 2-cycle losses stability and 4-cycle arises and is asymptotically stable
- $3.544 < r \leq 3.5699$: process of period doubling continues to periods $2^3, 2^4, \ldots$ and finally for $r \approx 3.5699$ chaos appears
- $3.5699 < r \leq 3.82$: transitions between periodical and chaotic behavior
- $3.82 < r \leq 3.85$: period 3 appears

- $3.85 < r < 4$: only chaotic behavior

Bifurcation diagram for logistic map, for r between 3.5 and 3.58 is given on the Figure 2.

Figure 2. Bifurcation diagram for logistic map (left) and enlarged part of bifurcation diagram (right), for r between 3.5 and 3.58



Source: Authors' calculations

Discrete dynamical system (8) is in the form similar to logistic map (9). In order to achieve the logistic map form, we must divide (8) with maximal value of the wheat price in the observed time series P_{max}. Denote $p_t = P_t/P_{max}$ and transform (8) to obtain:

$$p_{t+1} = p_t(1 + \theta(a + c) - \theta P_{\max}(b - d)p_t). \tag{10}$$

Now, in (10), $p_t \in [0,1]$. To transform (10) into (9) we need to introduce one more change of variables:

$$z_t = \frac{\theta P_{max}(b-d)}{1+\theta(a+c)} p_t. \tag{11}$$

If we made additional assumption that coefficients a, b, c, d, θ and P_{max} are such that after change of variables (11) variable $z_t \in [0,1]$, then (10) can finally be transformed into form of (9):

$$z_{t+1} = z_t\big(1 + \theta(a + c)\big)\left(1 - \frac{\theta P_{max}(b-d)}{1+\theta(a+c)} p_t\right) = \big(1 + \theta(a + c)\big)z_t(1 - z_t).$$

Since we briefly described well known behaviour of logistic map with dependence of parameter r (where $r = 1 + \theta(a + c)$), we will not get into the details here of the values where period doubling happens. We just mention that for $r \approx 3.835$ cycle of period three appears, and then the chaos appears, see Li and Yorke, 2004; famous paper.

Appendix B

We suggest non-linear supply-demand model instead of linear supply-demand model, with appropriate equations:

$$D_t = a - bP_t^q D_t = a - bP_t^q, S_t = -c - dP_t^q$$

where $a, b > 0$; $c, d < 0$ and $q > 0$ is arbitrary parameter. In order to obtain chaotic wheat producer price growth model, we suppose that relative change in wheat price is proportional with surplus of demand for wheat: $\frac{P_{t+1} - P_t}{P_t} = \theta(D_t - S_t)$. Further, we obtain: $P_{t+1} = P_t\left(1 + \theta(a + c) - \theta(b - d)P_t^q\right)$.

Again, in order to norm the price, we divide previous equation by P_{max}, denote $p_t = P_t / P_{max}$, and obtain second model:

$$p_{t+1} = p_t\left(1 + \theta(a + c) - \theta P_{max}^q (b - d)p_t^q\right) = \alpha p_t - \beta p_t^\gamma .$$

Finally, we introduce the change of variables: $z_t = \left(\frac{\theta P_{max}^q (b-d)}{1+\theta(a+c)}\right)^{1/q} p_t$, and from previous equation we obtain:

$$\left(\frac{\theta P_{max}^q (b-d)}{1+\theta(a+c)}\right)^{1/q} p_{t+1} =$$

$$\left(\frac{\theta P_{max}^q (b-d)}{1+\theta(a+c)}\right)^{1/q} p_t(1 + \theta(a + c))\left(1 - \left(\left(\frac{\theta P_{max}^q (b-d)}{1+\theta(a+c)}\right)^{1/q} \cdot p_t\right)^q\right)$$

.With change of variables, previous relation is equivalent to

$$z_{t+1} = \left(1 + \theta(a + c)\right)z_t\left(1 - z_t^q\right) = rz_t(1 - z_t^q)$$

what is in the form of the generalized logistic equation.